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THE REFORMULATED FIRST ZAGREB INDEX OF THE LINE GRAPHS OF THE SUBDIVISION GRAPH FOR CLASS OF GRAPHS

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ABSTRACT

The reformulated first Zagreb index is the edge version of first Zagreb index of chemical graph theory. The aim of this paper is to obtain an expression for the reformulated first Zagreb index of the some class of graphs such as Tadpole graph, Wheel graph, Ladder graph. Further we also obtain the reformulated first Zagreb index of the line graph, subdivision graph and line graph of subdivision graph for class of graphs.

KEYWORDS: Tadpole graph, Wheel graph, Ladder graph, Line graph, Subdivision graph, Reformulated first Zagreb index.

INTRODUCTION

Different topological indices are found to be usefull in isomer discrimination, structure-property relationship, structure-activity relationship, pharamaceutical drug design, etc. in chemistry, biochemistry and nanotechnology. Suppose G is a simple connected graph and $V(G)$ and $E(G)$ respectively denote the vertex set and edge set of G. Let, for any vertex $v \in V(G)$, deg(v) denotes its degree, that is the number of neighbors of v. Let n-vertices of G be denoted by $v_1, v_2, \ldots v_n$. If the edge of G are $(v_1, v_2)(v_2, v_3),\ldots,(v_{n-2}, v_{n-1})(v_{n-1}, v_n)$. Then the graph is called a path graph and is denoted by P_n. The first and second Zagreb indices of a graph, denoted by $M_1(G)$ and $M_2(G)$ are among the oldest, most popular and most extensively studied vertex-degree-based topological indices. These indices were introduced Gutman and Trinajstić in 1972 [7] to study the structure-dependency of the total π-electron energy $(\varepsilon)$and are respectively defined as

$$M_1(\mathrm{G}) = \sum_{v\in \mathrm{V(G)}} \deg(v)^2 = \sum [\deg(\mathrm{u}) + \deg(v)] \tag{1}$$

and

$$M_2(G) = \sum_{uv\in E(G)} [\deg(u)\deg(v)] \tag{2}$$

These indices are extensively studied in (chemical) graph theory. Interested readers are referred to[17, 2] for some recent reviews on the topic. Milićević *et al*. [12] reformulated the Zagreb indices interms of edge degrees instead of vertex degrees, where the degree of an edge $e = uv$ is defined as deg(e) = deg(u) + deg(v) - 2. Thus, the reformulated first and second Zagreb indices of a graph G are defined as

$$EM_1(G) = \sum_{e\in \mathrm{E(G)}} \deg(e)^2 \tag{3}$$

and

$$EM_2(G) = \sum_{e\sim f} [\deg(e)\ \deg(f)] \tag{4}$$

where $e \sim f$ means that the edges e and f share a comman vertex in G, i.e., they are adjacent. Different mathematical properties of reformulated Zagreb indices have been studied in [18]. In [8]. Ilić *et al.*, establish further mathematical properties of the reformulated Zagreb indices. In [15], bounds for the reformulated first Zagreb index of graphs with connectivity at most k are obtained. De [4] found some upper and lower bounds of these indices interm of some other graph invariants and also derived reformulated Zagreb indices of a class of dendrimers [3]. Ji *et al*. [9, 10] computed these indices for acyclic, unicyclic, bicyclic and tricyclic graphs.

PRILIMINARIES

Graph operations play a very important rule in mathematical chemistry, since some chemically interesting graphs can be obtained from some simpler graphs by different graph operations. In [11], Khalifeh *et al*., derived some exact expressions for computing first and second Zagreb indices of some graph operations. Ashrafi *et al*. [1] derived explicit expressions for Zagreb coindices of different graph operations.

Recently, there has been some interest in subdivision associated with Zagreb indices [14]. The fact that many interesting graphs are composed of simpler graphs that serve as as their basic building blocks prompted interest in the type of a relationship between the Zagreb index of a composite graph and Zagreb index of its building blocks. We refer the reader to [13] for the proof of this fact and for more information on Zagreb indices. Obviously, the Zagreb indices can be viewed as contributions of pairs of adjacent vertices to the vertex-weighted wiener number [6].

The Subdivision graph $S(G)$ is the graph obtained from G by replacing each of its edge by a path of length 2, or equivalently, by inserting an additional vertex into each edge of G. The Line graph of the graph G, written $L(G)$, is the simple graph whose vertices are the edges of G, with $e,f \in E(L(G))$ when e and f have a common end point in G. The $T_{n,k}$ Tadpole graph [16] is the graph obtained by joining a cycle graph C_n to a path of length k. The Ladder graph L_n is given by $L_n = K_2 \times P_n$, where P_n is a path. It is therefore equivalent to the grid graph $G_{2,n}$. The graph obtained via this definition has the advantage of looking like a Ladder, having two rails and n rungs between them. The Wheel graph denoted by W_n, is obtained by adding a new vertex to the cycle C_n and connects this new vertex to each vertex of C_n.

In this paper we present some exact expressions for the reformulated first Zagreb index of the some class of graphs such as Tadpole Graph, Wheel Graph and Ladder Graph. Also we obtain the reformulated first Zagreb index of the line graph and line graph of subdivision graph for above mentioned class of graphs.

The following theorem is useful in the further result.

Theorem A. [5]. The Ladder graph L_n made by n square and (2n + 2 vertices is the cartesian product of P_2 and P_{n+1}, so the reformulated first Zagreb index of L_n is given by $EM_1[L_n] = 48n - 36.$

RESULTS

In this section, we derive expressions for the reformulated first Zagreb index of the line graph and line graph of subdivision graph for Tadpole Graph, Wheel Graph and Ladder Graph.

Theorem 3.1. For the Tadpole graph ($T_{n,k}$) the reformulated first Zagreb index is $EM_1(T_{n,k}) = 4(n+k)+12,\ \forall k > 1.$

Proof. The Tadpole graph consists of a cycle C_n with n lines and a path P_k of length k.

(i) The cycle C_n of a Tadpole graph consists of $(n - 2)$ edges of degree 2 and the remaining two edges of degree 3.
From equation (3),
The reformulated first Zagreb index for a cycle of a Tadpole graph is,

$$\sum_{e \in E(C_n)} \deg(e)^2 = [(n-2)\times 2^2] + (2\times 3^2) \quad (5)$$

(ii) The path P_k of a Tadpole graph consists of $(k - 2)$ edges of degree 2, one edge of degree 3 and one pendent edge of degree one.
From equation (3),
The reformulated first Zagreb index for a path of a Tadpole graph is,

$$\sum_{e \in E(P_k)} \deg(e)^2 = [(k-2)\times 2^2] + (1\times 3^2) + (1\times 1^2) \quad (6)$$

From (5) and (6),

$$EM_1(T_{n,k}) = [((n-2)\times 2^2) + (2\times 3^2)] + [((k-2)\times 2^2) + (1\times 3^2) + (1\times 1^2)]$$

On simplification,

$$EM_1(T_{n,k}) = 4(n+k)+12,\ \forall k > 1.$$

Theorem 3.2. For the line graph of a Tadpole graph $L(T_{n,k})$ the reformulated first Zagreb index is

$$EM_1[L(T_{n,k})] = 4(n+k)+52, \forall k > 2.$$

Proof. The line graph of a Tadpole graph consists of $(n + k – 6)$ edges of degree 2, three edges of degree 3, three edges of degree 4 and one pendent edge of degree one, which is contributed in the following way i.e.,

(i) The cycle C_n of $L(T_{n,k})$ consists of $(n – 3)$ edges of degree 2, two edges of degree 3 and one edge of degree 4.
From equation (3),
The reformulated first zagreb index for a cycle of a $L(T_{n,k})$ is,

$$\sum_{e\in E(C_n)} \deg(e)^2 = ((n-3)\times 2^2) + (2\times 3^2) + (1\times 4^2) \tag{7}$$

(ii) The path P_k corresponding to the path in $T_{n,k}$ in $L(T_{n,k})$ consists of $(k – 3)$ edges of degree 2, two edges of degree 4, one edge of degree 3 and one pendent edge of degree one.
From equation (3),
The reformulated first Zagreb index for a path of a $L(T_{n,k})$ is,

$$\sum_{e\in E(P_k)} \deg(e)^2 = ((k-3)\times 2^2) + (2\times 4^2) + (1\times 3^2) + (1\times 1^2) \tag{8}$$

From (7) and (8),

$$EM_1[L(T_{n,k})] = [((n-3)\times 2^2)+(2\times 3^2)+(1\times 4^2)]+[((k-3)\times 2^2)+(2\times 4^2)+(1\times 3^2)+(1\times 1^2)]$$

On simplification,

$$EM_1[L(T_{n,k})] = 4(n+k)+52,\ \forall k > 2.$$

Theorem 3.3. For the subdivision graph of a Tadpole graph $S(T_{n,k})$ the reformulated first Zagreb index is

$$EM_1[S(T_{n,k})] = 8(n+k)+12,\ \forall k \geq 1.$$

Proof. The cycle of a subdivision graph of the Tadpole graph $S(T_{n,k})$ contains $2n$ edges. Similarly the path of $S(T_{n,k})$ contains $2k$ edges.

(i) The cycle C_{2n} of $S(T_{n,k})$ consists of two edges of degree 3 and remaining $(2n – 2)$ edges of degree 2.
From equation (3),
The reformulated first Zagreb index for a cycle of a $S(T_{n,k})$ is,

$$\sum_{e\in E(C_{2n})} \deg(e)^2 = (2\times 3^2) + ((2n-2)\times 2^2) \tag{9}$$

(ii) The path P_{2k} of $S(T_{n,k})$ consists of one edge of degree 3, one pendent edge of degree one and remaining $(2k – 2)$ edges of degree 2.
From equation (3),
The reformulated first Zagreb index for a path of a $S(T_{n,k})$ is,

$$\sum_{e\in E(P_{2k})} \deg(e)^2 = (1\times 3^2) + (1\times 1^2) + ((2k-2)\times 2^2) \tag{10}$$

From (9) and (10),

$$EM_1[S(T_{n,k})] = [(2\times 3^2)+((2n-2)\times 2^2)]+[(1\times 3^2)+(1\times 1^2)+((2k-2)\times 2^2)]$$

On simplification,

$$EM_1[S(T_{n,k})] = 8(n+k)+12, \forall k \geq 1.$$

Theorem 3.4. For the line graph of a subdivision graph of a Tadpole graph $L(S(T_{n,k}))$ the reformulated first zagreb index is

$$EM_1[L(S(T_{n,k}))] = 8(n+k)+52, \forall k \geq 1.$$

Proof. The line graph of a subdivision graph of a Tadpole graph contains $(2n + 2k – 6)$ edges of degree 2, three edges of degree 3, three edges of degree 4 and one pendent edge of degree one,which is contributed in the following way i.e.,

(i) The cycle C_{2n} of $L(S(T_{n,k}))$ consists of $(2n – 3)$ edges of degree 2, two edges of degree 3 and one edge of degree 4.
From equation (3),
The reformulated first Zagreb index for a cycle of a $L(S(T_{n,k}))$ is,

$$\sum_{e\in E(C_{2n})} \deg(e)^2 = ((2n-3)\times 2^2)+(2\times 3^2)+(1\times 4^2) \qquad (11)$$

(ii) The path P_{2k} corresponding to the path in $S(T_{n,k})$ in $L(S(T_{n,k}))$ consists of $(2k – 3)$ edges of degree 2, two edges of degree 4, one edge of degree 3 and one pendent edge of degree one.
From equation (3),
The reformulated first Zagreb index for a path of a $L(S(T_{n,k}))$ is,

$$\sum_{e\in E(P_{2k})} \deg(e)^2 = ((2k-3)\times 2^2)+(2\times 4^2)+(1\times 3^2)+(1\times 1^2) \qquad (12)$$

From (11) and (12),

$$EM_1[L(S(T_{n,k}))]=[((2n-3)\times 2^2)+(2\times 3^2)+(1\times 4^2)]+[((2k-3)\times 2^2)+(2\times 4^2)+(1\times 3^2)+(1\times 1^2)]$$

On simplification,

$$EM_1[L(S(T_{n,k}))]=8(n+k)+52, \forall k\geq 1.$$

Theorem 3.5. For the Wheel graph W_n the reformulated first Zagreb index is $EM_1(W_n)=(n-k)\times(n^2+16).$

Proof. The Wheel graph W_n with n number of vertices. The Wheel graph consists of $(n – 1)$ edges of degree 4 and $(n – 1)$ edges of degree n.
From equation (3),
The reformulated first Zagreb index for Wheel graph is,

$$EM_1(W_n)=((n-1)\times 4^2)+((n-1)\times n^2)$$

On simplification,

$$EM_1(W_n)=(n-k)\times(n^2+16).$$

Theorem 3.6. For the line graph of a Wheel graph $L(W_n)$ the reformulated first Zagreb index is

$$EM_1[L(W_n)]=(n-1)\times[2[(n-1)^2(n-2)+(n+2)^2]+36].$$

Proof. The line graph of a Wheel graph consists of $(n – 1)$ edges of degree 6, $(2n – 2)$ edges of degree $(n + 2)$ and exactly one complete graph with $(n – 1)$ vertices i.e., $\frac{(n-1)(n-2)}{2}$ edges of degree $(2n – 2)$.

From equation (3),
The reformulated first Zagreb index for line graph of a Wheel graph is,

$$EM_1[L(W_n)]=((n-1)\times 6^2)+((2n-2)\times(n+2)^2)+[\frac{(n-1)(n-2)}{2}]\times(2n-2)^2$$

On simplification,

$$EM_1[L(W_n)]=(n-1)\times[2[(n-1)^2(n-2)+(n+2)^2]+36].$$

Theorem 3.7. For the subdivision graph of the Wheel graph $S(W_n)$ the reformulated first Zagreb index is

$$EM_1[S(W_n)]=(n-1)\times[3^3+(n-1)^2].$$

Proof. The subdivision graph of a Wheel graph consists of $3(n – 1)$ edges of degree 3 and $(n – 1)$ edges of degree $(n – 1)$.
From equation (3),
The reformulated first Zagreb index for subdivision graph of the Wheel graph $S(W_n)$ is,

$$EM_1[S(W_n)]=[3(n-1)\times 3^2]+[(n-1)\times(n-1)^2]$$

On simplification,

$$EM_1[S(W_n)]=(n-1)\times[3^3+(n-1)^2].$$

Theorem 3.8. For the line graph of a subdivision graph of the Wheel graph $L(S(W_n))$ the reformulated first Zagreb index is

$$EM_1[L(S(W_n))] = (n-1)\times[n^2 + 2(n-2)^3 + 64].$$

Proof. The line graph of a subdivision graph of the Wheel graph $L[S(W_n)]$ consist of $4(n – 1)$ edges of degree 4, $(n – 1)$ edges of degree n and exactly one complete graph with $(n – 1)$ vertices i.e., $\frac{(n-1)(n-2)}{2}$ edges of degree $(2n-2)$.

From equation (3),

The reformulated first Zagreb index for $L[S(W_n)]$ is,

$$EM_1[L(S(W_n))] = [4(n-1)\times 4^2) + [(n-1)\times n^2] + [\frac{(n-1)(n-2)}{2}]\times(2n-2)^2$$

On simplification,

$$EM_1[L(S(W_n))] = (n-1)\times[n^2 + 2(n-2)^3 + 64].$$

Theorem 3.9. For the Ladder graph $L_n=K_2 \times P_n$ the reformulated first Zagreb index is $EM_1(L_n) = 12(4n-7),\ \forall n > 2.$

Proof. The proof follows from Theorem A and by replacing n by $(n - 1)$.

Theorem 3.10. For the line graph of a Ladder graph $L(L_n)$ the reformulated first Zagreb index is

$$EM_1[L(L_n)] = 4(54n-121),\ \forall n > 3.$$

Proof. The line graph of a Ladder graph consist of four edges of degree 3, eight edges of degree 5 and $(6n – 20)$ edges of degree 6.

From equation (3),

The reformulated first Zagreb index for the line graph of ladder graph $L(L_n)$ is,

$$EM_1[L(L_n)] = (4\times 3^2) + (8\times 5^2) + [(6n-20)\times 6^2]$$

On simplification,

$$EM_1[L(L_n)] = 4(54n-121),\ \forall n > 3.$$

Theorem 3.11. For the subdivision graph of a Ladder graph $S(L_n)$ the reformulated first Zagreb index is

$$EM_1[S(L_n)] = 2(27n-38),\ \forall n > 1.$$

Proof. The subdivision graph of a Ladder graph consists of eight edges of degree 2 and $6(n – 2)$ edges of degree 3.

From equation (3),

The reformulated first Zagreb index for subdivision graph of a Ladder graph $S(L_n)$ is,

$$EM_1[S(L_n)] = (8\times 2^2) + [6(n-2)\times 3^2]$$

On simplification,

$$EM_1[S(L_n)] = 2(27n-38),\ \forall n > 1.$$

Theorem 3.12. For the line graph of a subdivision graph of the Ladder graph $L(S(L_n))$ the reformulated first Zagreb index is

$$EM_1[L(S(L_n))] = 4(36n-65),\ \forall n > 2.$$

Proof. The line graph of a subdivision graph of the ladder graph $L[S(L_n)]$ consist of six edges of degree 2, four edges of degree 3 and $(9n – 20)$ edges of degree 4.

From equation (3),

The reformulated first Zagreb index for $L(S(L_n))$ is,

$$EM_1[L(S(L_n))] = (6\times 2^2) + (4\times 3^2) + [(9n-20)\times 4^2]$$

On simplification,

$$EM_1[L(S(L_n))] = 4(36n-65),\ \forall n > 2.$$

CONCLUSION

In this paper, we studied the reformulated first Zagreb index, which is also called as edge version of a first Zagreb index and have calculated the reformulated first Zagreb index of some class of graphs. Nevertheless, there are still many other class of graphs that are not covered here. For further research, the second reformulated Zagreb index for other class of graphs can be computed.

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